Macroeconomics B 2023/24

Final Exam

June 12-14 2024

Instructions:

- You have 48 hours to work on the exam: From Wednesday June 12 at 12 pm (noon) until **Friday June 14 at 12.00 pm (noon)**. Submissions are via Digital Exam.
- There are three problems, which can be found below. In addition, the exam includes the file `us_data.xlsx`, which contains the data for problem 2.
- All questions of all problems must be answered.

Submissions:

- Your answers must be in English
- You should submit:
  1. A PDF with your answers to Problems 1, 2 and 3.
  2. An Excel spread sheet or a Jupyter notebook with the calculations for Problem 2.
  3. A Jupyter notebook with your calculations for Problem 3.
- All files submitted should include your **exam number** (and *not* your name).
- The submitted Jupyter notebook(s) should be able to run directly without errors.

During the exam:

- All aids are allowed.
- Your are not allowed to communicate with anyone about the exam.
- In case of practical questions during the exam please contact the study administration (uddannelse@diku.dk).
  In case of other questions, please contact the course coordinator (mbta@econ.ku.dk)

Problem 1. Monetary Policy With Measurement Error

Consider the following model of a closed economy in usual notation

$$y_t - \bar{y} = -\alpha_2 (r_t - \bar{r}) + v_t, \quad \alpha_2 > 0 \tag{1}$$

$$r_t \equiv i_t - \pi^e_{t+1,t} \tag{2}$$

$$i_t = \bar{r} + \pi^e_{t+1,t} + h(\pi_t - \pi^*) + b(y_t + \varepsilon_t - \bar{y}) + \hat{\rho}_t, \quad h, b > 0 \tag{3}$$

$$\pi_t = \pi^e_{t,t-1} + \gamma(y_t - \bar{y}) \tag{4}$$

The second index/subscript on expected inflation denotes when expectation was formed. v_t reflects shocks to consumer and business confidence, $\hat{\rho}_t$ denotes the part of the relevant long-term nominal interest rate that is beyond the control of the central bank, and ε_t denotes a measurement error in the value of real output used in the monetary policy rule.

1. Explain the economics behind relations eq. (1) - eq. (4).
2. Derive an expression for the AD-kurve of the economy. Explain in economic terms how the slope of the AD curve is affected by the values of the parameters α_2, h and b.

Assume that expectations are static, i.e. specifically that

$$\pi^e_{t,t-1} = \pi_{t-1} \tag{5}$$

It is assumed that the economy has been in a long-run equilibrium in the absence of shocks for a number of periods, when, in period 1 and 2 only, there is a temporary positive measurement error in output, $\varepsilon_1 = \varepsilon_2 = \varepsilon > 0$, $\varepsilon_t = 0$ for all $t \neq 1, t \neq 2$.

3. Provide a graphical analysis and an economic explanation of how the economy is affected in periods 1 and onwards.
4. Show that by defining $\hat{y}_t \equiv y_t - \bar{y}$, eq. (1) - eq. (5) may, in the absence of shocks, be combined to yield $\hat{y}_t = \beta \hat{y}_{t-1}$ where β is an expression that depends on parameters of the model. Explain in economic terms how and why the the parameters α_2, h and b affect the speed of convergence towards long-run equilibrium.
5. Explain how it would change your analysis in question 3, both graphically and in economic terms, if instead of the measurement error, there were in periods 1 and 2 a temporary negative value of v_t, $v_1 = v_2 = v < 0$ or a temporary positive value of $\hat{\rho}_t$, $\hat{\rho}_1 = \hat{\rho}_2 = \hat{\rho} > 0$, $v_t = \hat{\rho}_t = 0, t \neq 1, t \neq 2$?

From now on it is assumed instead that expectations are rational, i.e. that eq. (5) is replaced by

$$\pi^e_{t,t-1} = E(\pi_t | I_{t-1}) \tag{6}$$

It is furthermore assumed that the ε_t, $\hat{\rho}_t$ and v_t have zero means

$$E(\varepsilon_t | I_{t-1}) = E(\hat{\rho}_t | I_{t-1}) = E(v_t | I_{t-1}) = 0 \tag{7}$$

6. Show that the equilibrium values of output and inflation are

$$y_t = \bar{y} + \frac{v_t - \alpha_2 b \varepsilon_t - \alpha_2 \hat{\rho}_t}{1 + \alpha_2 b + \gamma \alpha_2 h} \tag{8}$$

$$\pi_t = \pi^* + \gamma \frac{v_t - \alpha_2 b \varepsilon_t - \alpha_2 \hat{\rho}_t}{1 + \alpha_2 b + \gamma \alpha_2 h} \tag{9}$$

7. Does the *policy ineffectiveness proposition* hold in the present model? Why/why not?

8. Analyse once again, both graphically and in economic terms, the situation from question 3 where in periods 1 and 2 only there is a positive measurement error, $\varepsilon_1 = \varepsilon_2 = \varepsilon > 0$. What are the similarities and differences compared with question 3 from period 1 onwards?
Note: it is assumed that the measurement error in both periods is unanticipated, i.e. no measurement error in period 1 is expected in period 0 and no measurement error in period 2 is expected in period 1.

Assuming that ε_t, $\hat{\rho}_t$ and v_t are uncorrelated and letting σ^2_ε, σ^2_ρ and σ^2_ε denote their variances, it follows from eq. (8) and eq. (9) that

$$\sigma^2_y \equiv Var(y_t) = \frac{\sigma^2_v + \alpha_2^2 b^2 \sigma^2_\varepsilon + \alpha_2^2 \sigma^2_\rho}{(1 + \alpha_2 b + \gamma \alpha_2 h)^2} \tag{10}$$

$$\sigma^2_\pi \equiv Var(\pi_t) = \gamma^2 \frac{\sigma^2_v + \alpha_2^2 b^2 \sigma^2_\varepsilon + \alpha_2^2 \sigma^2_\rho}{(1 + \alpha_2 b + \gamma \alpha_2 h)^2} \tag{11}$$

Please note that σ^2_y and σ^2_π are proportional, $\sigma^2_\pi = \gamma^2 \sigma^2_\pi$. Hence minimising either σ^2_π or σ^2_y will minimise both.

9. Show that the value of b that solves the first order condition for minimising σ^2_y is:[1]

$$b = \frac{\alpha_2^2 \sigma^2_\rho + \sigma^2_v}{\alpha_2 \left(1 + h\gamma\alpha_2^2\right) \sigma^2_\varepsilon} \tag{12}$$

10. Explain why b depends on the values of h, σ^2_ε, σ^2_ρ and σ^2_v as can be seen from eq. (12).

Problem 2. Computing Taylor Rules for the US

In this exercise, you are asked to compute two different versions of a Taylor Rule for the US economy. To start with, you should assume that the Federal Reserve follows a Taylor rule of the type:

$$i_t = r^*_t + \pi_t + h(\pi_t - \pi^*) + b(y_t - \bar{y}), \quad h > 0, b > 0, \tag{13}$$

where i_t is the nominal interest rate; r^*_t is the natural interest rate in the economy; π^* is the Federal Reserve's inflation target; and $\bar{y}$ is the natural output of the economy. The parameters h and b

[1] You can use the fact that the second order condition for minimisation holds.

determines the Federal Reserve's response to inflation and output respectively. Please also hand in the Excel sheet or Jupyter notebook used to answer the question for grading.

Data. In the file `us_data.xlsx` you will find a dataset containing data from 1987Q1 to 2023Q4 and the following variables:

1. `r_star`: This is an estimate for the economy's natural interest rate (i.e. r_t^*).[2]
2. `pce`: This is the year-on-year percentage change in the personal consumption expenditures inflation index, which is the Federal Reserve's preferred measure of inflation. This variable corresponds to π_t in eq. (13).
3. `gdp`: This is the real GDP measured in billions of real 2017 dollars.
4. `ffr`: This is the effective federal funds rate in percent. This is the interest rate that the Federal Reserve controls through its monetary policy stance.

Questions. You are now asked to answer the following questions.

1. Use the data found in the spreadsheet to compute the inflation gap. You can assume that $\pi^* = 2\%$. Compute also the output gap $(y_t - \bar{y})$ in eq. (13). You should do this by using the HP-filter. Discuss the value of λ you choose when using the HP-filter. Report the computed output- and inflation gap in a plot.
   Hint: You should *not drop* any observations from the dataset, as you have done in the exercises in class.
2. Use eq. (13) to compute a Taylor interest rate for the US economy. Assume that $h = b = 0.5$. Plot your computed Taylor interest rate and the actual interest rate in the same plot. Comment on the fit between your Taylor interest rate and the actual interest rate.
3. You are now asked to compute a Taylor rule which includes interest rate smoothing. This means that the Taylor rule in each period is given by the formula:

$$i_t = \rho_i i_{t-1} + (1 - \rho_i)\left[r_t^* + \pi_t + h(\pi_t - \pi^*) + b(y_t - \bar{y})\right]. \tag{14}$$

   Hence, the Taylor rule is a weighted average of last period's Taylor rate and the current period's Taylor rate. You can assume that $\rho_i = 0.85$. Plot the computed Taylor rate in the same figure as the Taylor rate you computed previously.
4. In light or your results in 1) - 3), discuss the pros and cons of following the different versions of the Taylor rule in eq. (13) and eq. (14). In your discussion, you should pay particular attention to the Covid-19 period.

[2]This is the so-called Laubach-Williams natural rate of interest estimate. This estimate is obtained using various econometric techniques to estimate the unobserved r^*.

Problem 3. The Closed Economy with Adaptive Expectations

In this question, you are asked to analyze the effects of inflation expectations on output and inflation dynamics in a closed economy. You should answer the questions in a Jupyter notebook. Make sure you submit the notebook with your code for grading. The notebook should be able to run directly without errors.

Model. You are given the following model for a closed economy:

$$\pi_t = \pi_t^e + \gamma(y_t - \bar{y}) + s_t \tag{15}$$
$$y_t - \bar{y} = \alpha_1 (g_t - \bar{g}) - \alpha_2 (r_t - \bar{r}) - \alpha_3 (\tau_t - \bar{\tau}) + v_t \tag{16}$$
$$i_t = \bar{r} + \pi_{t+1}^e + h(\pi_t - \pi^*) + b(y_t - \bar{y}) + \hat{\rho}_t \tag{17}$$
$$r_t \equiv i_t - \pi_{t+1}^e \tag{18}$$
$$\pi_t^e = \phi \pi_{t-1}^e + (1-\phi)\pi_{t-1} \tag{19}$$

Using eq. (16), eq. (17) and eq. (18), it can be shown that the AD-curve can be written as:

$$y_t - \bar{y} = -a(\pi_t - \pi^*) + z_t, \quad a \equiv \frac{\alpha_2 h}{1 + \alpha_2 b}$$

Assume that both the demand and the supply disturbances are AR(1) processes:

$$z_t = \rho_z z_{t-1} + \sigma_{\epsilon_z} \epsilon_{z,t}, \quad \epsilon_{z,t} \sim \mathcal{N}(0,1) \tag{20}$$
$$s_t = \rho_s s_{t-1} + \sigma_{\epsilon_s} \epsilon_{s,t}, \quad \epsilon_{s,t} \sim \mathcal{N}(0,1) \tag{21}$$

with $\rho_z, \rho_s \in [0,1)$ and where $\epsilon_{z,t}$ is a demand shock, and $\epsilon_{s,t}$ is a supply shock that are independently normally distributed. When answering the questions, assume the following parameter values: $a = 0.4, \gamma = 0.1, \sigma_{\epsilon_z} = 1.0, \sigma_{\epsilon_s} = 0.2, \rho_z = 0.75, \rho_s = 0.15$.

Questions. You are now asked to answer the following questions.

1. Explain the equations in eq. (15) - eq. (19). Have a special focus on the meaning of eq. (19). Explain the difference between $\phi = 0$ and $\phi > 0$.
2. Show analytically that the AD-curve and AS-curve can be written as:

$$\hat{y}_t = -a\hat{\pi}_t + z_t$$
$$\hat{\pi}_t = \hat{\pi}_{t-1} + \gamma \hat{y}_t - \phi\gamma\hat{y}_{t-1} + s_t - \phi s_{t-1},$$

   where $\hat{y}_t \equiv y_t - \bar{y}$ and $\hat{\pi}_t \equiv \pi_t - \pi^*$.

It can be shown that the model can be reduced to the following stochastic, linear, first order difference equations in $\hat{y}_t$ and $\hat{\pi}_t$:

$$\hat{y}_t = d\hat{y}_{t-1} + \beta(z_t - z_{t-1}) - \alpha\beta s_t + \alpha\beta\phi s_{t-1}, \tag{22}$$
$$\hat{\pi}_t = d\hat{\pi}_{t-1} + \beta\gamma z_t - \beta\gamma\phi z_{t-1} + \beta s_t - \beta\phi s_{t-1}, \tag{23}$$
$$d \equiv \frac{1 + a\gamma\phi}{1 + a\gamma} < 1, \quad \beta \equiv \frac{1}{1 + a\gamma} < 1.$$

Note you are *not* asked to derive these.

3. In Python, code up the functions in eq. (22) and eq. (23).

4. You will now construct impulse responses to answer the following question: *Starting from the long-run equilibrium, how does the economy evolve in response to a one-time, one standard-deviation negative (that is, inflationary) supply shock?* You should calculate the impulse response functions for y_t, π_t and π_t^e and for both $\phi = 0$ and $\phi = 0.5$. Plot the impulse responses for 75 periods. Explain the intuition behind your results. Why is the response different for $\phi = 0$ and $\phi = 0.5$?
   Hint: Remember that the parameter d depends on ϕ, and it thus needs to be recomputed when ϕ is changed.

5. Continue to make the same assumptions about the shock processes as in the last question. But now instead of computing impulse responses (a one-time shock) simulate the AS-AD model for 200 periods (drawing a realization of $\epsilon_{z,t}$ and $\epsilon_{s,t}$ for each of these periods). Please use the seed "2023" for comparability. Plot the resulting time series and calculate the following statistics using `std` and `corrcoef` from `numpy`:

   (a) Standard deviation of $\hat{y}_t$, $std(\hat{y}_t)$
   (b) Standard deviation of $\hat{\pi}_t$, $std(\hat{\pi}_t)$
   (c) Correlation between $\hat{y}_t$ and $\hat{\pi}_t$, $corr(\hat{y}_t, \hat{\pi}_t)$
   (d) Auto-correlation between $\hat{y}_t$ and $\hat{y}_{t-1}$, $corr(\hat{y}_t, \hat{y}_{t-1})$
   (e) Auto-correlation between $\hat{\pi}_t$ and $\hat{\pi}_{t-1}$, $corr(\hat{\pi}_t, \hat{\pi}_{t-1})$

   Perform the simulation for both $\phi = 0.0$ and $\phi = 0.5$. Report results with 4 decimal points.

6. Now you are asked to compute the social loss, $\mathbb{L}$. To compute the total social loss, you should use the formula:

$$\mathbb{L} = \sum_{\tau=0}^{200} \delta^{\tau} \left(SL_{t+\tau} \right), \tag{24}$$

   where

$$SL_t = (y_t - \bar{y})^2 + \kappa \left(\pi_t - \pi^* \right)^2 ,$$

   where δ is a discount factor that discounts future social losses and κ is the social weight on inflation fluctuations relative to output fluctuations. When you compute the total social loss, you can assume that $\delta = 0.96$ and $\kappa = 1$. Calculate the social loss for both $\phi = 0.0$ and $\phi = 0.5$. Report results with 4 decimal points.

7. You are now asked to calibrate the value of ϕ. In the calibration, you should target a standard deviation of the inflation gap of 0.52, which corresponds the standard deviation of US inflation from 1947Q1 to 2017Q1. Report the calibrated value of ϕ.

We will now change the formulation of inflation expectations. Expectations are now given by:

$$\pi_t^e = \phi \pi^* + (1 - \phi) \pi_{t-1}. \tag{25}$$

This means that the inflation expectations are now given as weighted average of the central bank's inflation target π^* and last period's inflation, π_{t-1}. It can be shown that the model can then be written as:

$$\hat{y}_t = (1-\phi)\beta\hat{y}_{t-1} + \beta\left(z_t - (1-\phi)z_{t-1}\right) - a\beta s_t, \tag{26}$$
$$\hat{\pi}_t = \beta(1-\phi)\hat{\pi}_{t-1} + \gamma\beta z_t + \beta s_t, \tag{27}$$

where β is as defined above.

8. You are now asked to do the same simulation as you did in question 5, but using eq. (26) and eq. (27) instead. Perform the simulation for $\phi = 0$ and $\phi = 0.5$. Compute the same summary statistics as you did in question 5 and the social loss as in question 6. Compare the results and explain why/why not they are different.

9. Consider the following quote from Federal Reserve Chair Jerome Powell in 2023:[3]

   > Supply shocks that drive inflation high enough for long enough can affect the longer-term inflation expectations (...). Monetary policy must forthrightly address any risks of a potential de-anchoring of inflation expectations (...)

   Discuss this quote in light of your findings in the previous questions. If central banks are seeking to maximize social welfare, does the AS/AD-model then support a statement as the one made by Jerome Powell above? Explain why/why not.

[3]Link: https://www.federalreserve.gov/newsevents/speech/powell20231109a.htm